Spring 2012

EE 330

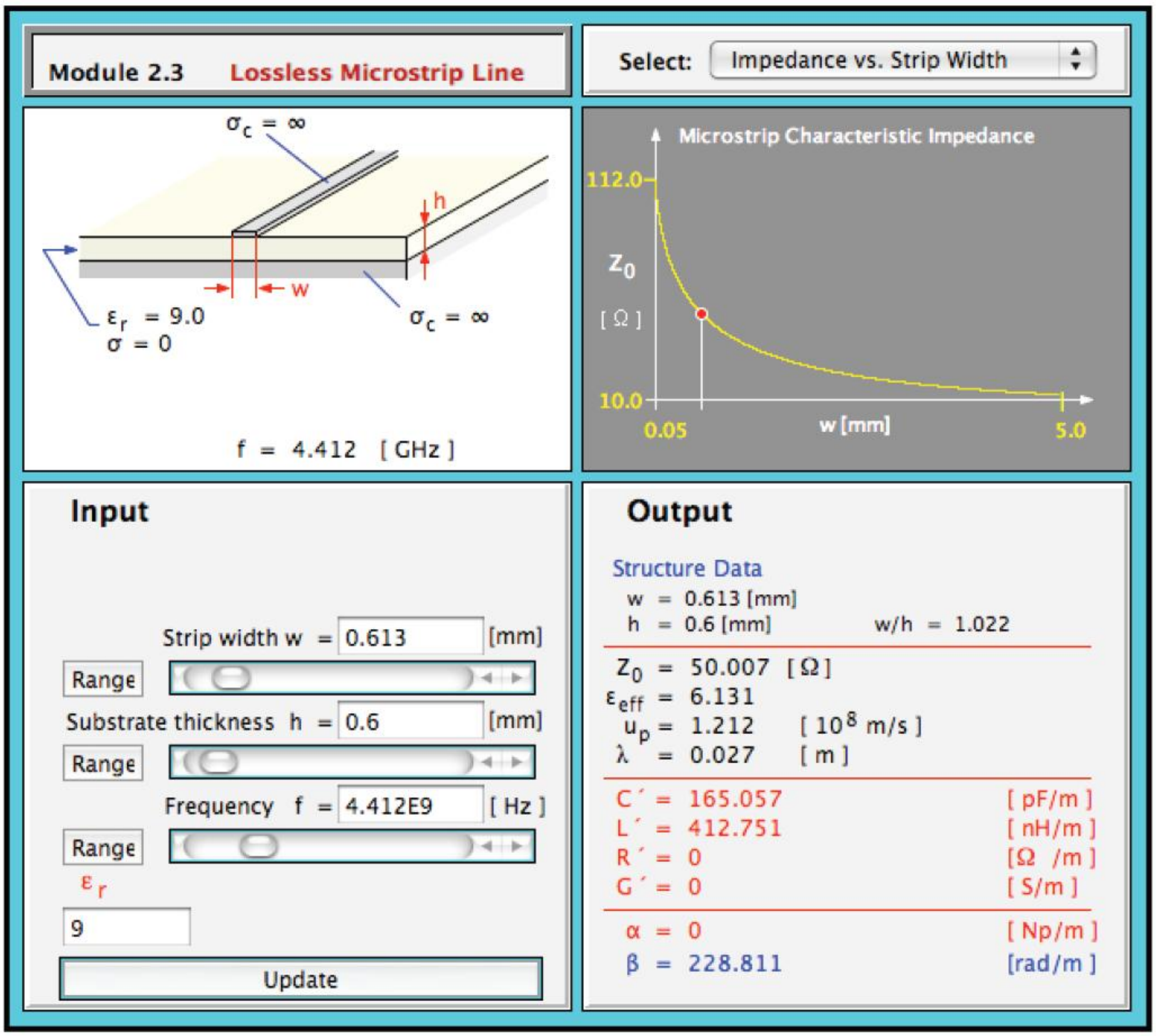
ENGINEERING ELECTROMAGNETICS

HW 3: *Due Friday, 27 January*
2.11, 2.13, 2.14, 2.17, 2.19, 2.27, 2.31, 2.33, 2.34, 2.40, 2.44, 2.47, 2.53

Problem 2.11 A 50-Ω microstrip line uses a 0.6-mm alumina substrate with $\varepsilon_r = 9$. Use CD Module 2.3 to determine the required strip width w. Include a printout of the screen display.

Solution: According to the solution provided by CD Module 2.3, the required strip width is

$$w = 0.613 \text{ mm}.$$



Problem 2.13 In addition to not dissipating power, a lossless line has two important features: (1) it is dispertionless (μ_p is independent of frequency) and (2) its characteristic impedance Z_0 is purely real. Sometimes, it is not possible to design a transmission line such that $R' \ll \omega L'$ and $G' \ll \omega C'$, but it is possible to choose the dimensions of the line and its material properties so as to satisfy the condition

$$R'C' = L'G' \quad \text{(distortionless line)}.$$

Such a line is called a *distortionless* line because despite the fact that it is not lossless, it does nonetheless possess the previously mentioned features of the loss line. Show that for a distortionless line,

$$\alpha = R'\sqrt{\frac{C'}{L'}} = \sqrt{R'G'}, \qquad \beta = \omega\sqrt{L'C'}, \qquad Z_0 = \sqrt{\frac{L'}{C'}}.$$

Solution: Using the distortionless condition in Eq. (2.22) gives

$$\begin{aligned}
\gamma = \alpha + j\beta &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{G'}{C'} + j\omega\right)} \\
&= \sqrt{L'C'}\sqrt{\left(\frac{R'}{L'} + j\omega\right)\left(\frac{R'}{L'} + j\omega\right)} \\
&= \sqrt{L'C'}\left(\frac{R'}{L'} + j\omega\right) = R'\sqrt{\frac{C'}{L'}} + j\omega\sqrt{L'C'}.
\end{aligned}$$

Hence,

$$\alpha = \mathfrak{Re}(\gamma) = R'\sqrt{\frac{C'}{L'}}, \qquad \beta = \mathfrak{Im}(\gamma) = \omega\sqrt{L'C'}, \qquad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}}.$$

Similarly, using the distortionless condition in Eq. (2.29) gives

$$Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}\sqrt{\frac{R'/L' + j\omega}{G'/C' + j\omega}} = \sqrt{\frac{L'}{C'}}.$$

Problem 2.14 For a distortionless line [see Problem 2.13] with $Z_0 = 50\ \Omega$, $\alpha = 20$ (mNp/m), $u_p = 2.5 \times 10^8$ (m/s), find the line parameters and λ at 100 MHz.

Solution: The product of the expressions for α and Z_0 given in Problem 2.6 gives

$$R' = \alpha Z_0 = 20 \times 10^{-3} \times 50 = 1 \quad (\Omega/\text{m}),$$

and taking the ratio of the expression for Z_0 to that for $u_p = \omega/\beta = 1/\sqrt{L'C'}$ gives

$$L' = \frac{Z_0}{u_p} = \frac{50}{2.5 \times 10^8} = 2 \times 10^{-7}\ (\text{H/m}) = 200 \quad (\text{nH/m}).$$

With L' known, we use the expression for Z_0 to find C':

$$C' = \frac{L'}{Z_0^2} = \frac{2 \times 10^{-7}}{(50)^2} = 8 \times 10^{-11}\ (\text{F/m}) = 80 \quad (\text{pF/m}).$$

The distortionless condition given in Problem 2.6 is then used to find G'.

$$G' = \frac{R'C'}{L'} = \frac{1 \times 80 \times 10^{-12}}{2 \times 10^{-7}} = 4 \times 10^{-4}\ (\text{S/m}) = 400 \quad (\mu\text{S/m}),$$

and the wavelength is obtained by applying the relation

$$\lambda = \frac{\mu_p}{f} = \frac{2.5 \times 10^8}{100 \times 10^6} = 2.5\ \text{m}.$$

Problem 2.17 Using a slotted line, the voltage on a lossless transmission line was found to have a maximum magnitude of 1.5 V and a minimum magnitude of 0.6 V. Find the magnitude of the load's reflection coefficient.

Solution: From the definition of the Standing Wave Ratio given by Eq. (2.73),

$$S = \frac{|\widetilde{V}|_{\max}}{|\widetilde{V}|_{\min}} = \frac{1.5}{0.6} = 2.5.$$

Solving for the magnitude of the reflection coefficient in terms of S, as in Example 2-5,

$$|\Gamma| = \frac{S-1}{S+1} = \frac{2.5-1}{2.5+1} = 0.43.$$

Problem 2.19 A 50-Ω lossless transmission line is terminated in a load with impedance $Z_L = (30 - j50)\ \Omega$. The wavelength is 8 cm. Find:

(a) the reflection coefficient at the load,

(b) the standing-wave ratio on the line,

(c) the position of the voltage maximum nearest the load,

(d) the position of the current maximum nearest the load.

(e) Verify quantities in parts (a)–(d) using CD Module 2.4. Include a printout of the screen display.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(30 - j50) - 50}{(30 - j50) + 50} = 0.57e^{-j79.8^\circ}.$$

(b) From Eq. (2.73),

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.57}{1 - 0.57} = 3.65.$$

(c) From Eq. (2.70)

$$d_{\text{max}} = \frac{\theta_r \lambda}{4\pi} + \frac{n\lambda}{2} = \frac{-79.8^\circ \times 8 \text{ cm}}{4\pi} \frac{\pi \text{ rad}}{180^\circ} + \frac{n \times 8 \text{ cm}}{2}$$
$$= -0.89 \text{ cm} + 4.0 \text{ cm} = 3.11 \text{ cm}.$$

(d) A current maximum occurs at a voltage minimum, and from Eq. (2.72),

$$d_{\text{min}} = d_{\text{max}} - \lambda/4 = 3.11 \text{ cm} - 8 \text{ cm}/4 = 1.11 \text{ cm}.$$

(e) The problem statement does not specify the frequency, so in Module 2.4 we need to select the combination of f and ε_r such that $\lambda = 5$ cm. With ε_r chosen as 1,

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-2}} = 3.75 \text{ GHz}.$$

The generator parameters are irrelevant to the problem.

The results listed in the output screens are very close to those given in parts (a) through (d).

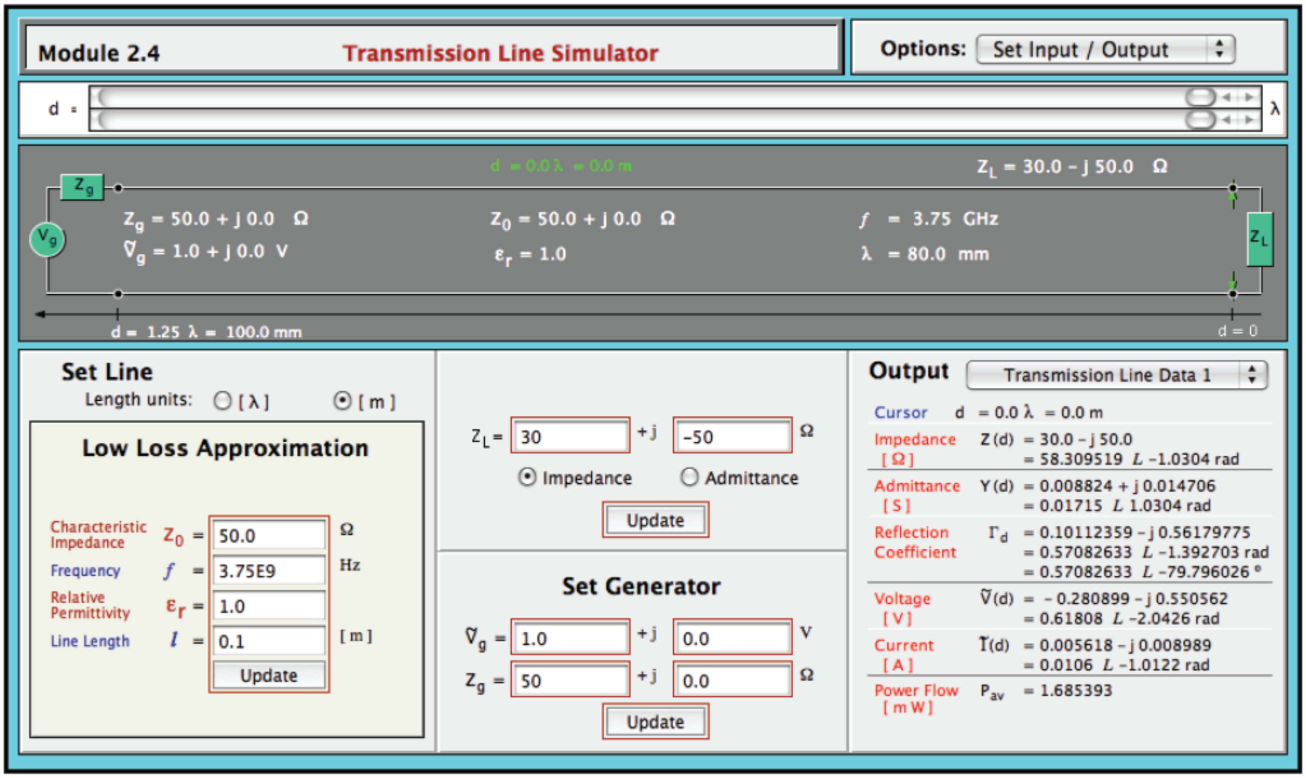


Figure P2.19(a)

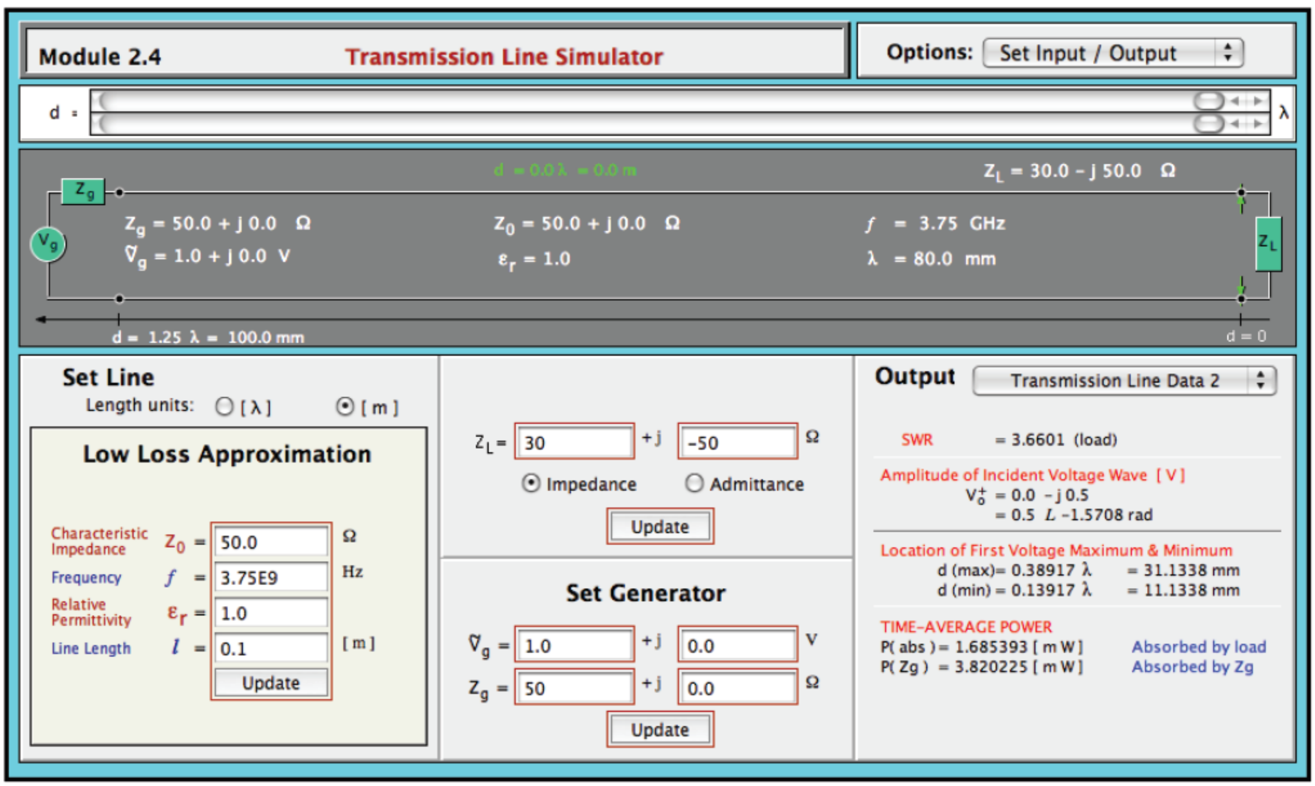


Figure P2.19(b)

Problem 2.27 At an operating frequency of 300 MHz, a lossless 50-Ω air-spaced transmission line 2.5 m in length is terminated with an impedance $Z_L = (40 + j20)$ Ω. Find the input impedance.

Solution: Given a lossless transmission line, $Z_0 = 50$ Ω, $f = 300$ MHz, $l = 2.5$ m, and $Z_L = (40 + j20)$ Ω. Since the line is air filled, $u_p = c$ and therefore, from Eq. (2.48),

$$\beta = \frac{\omega}{u_p} = \frac{2\pi \times 300 \times 10^6}{3 \times 10^8} = 2\pi \text{ rad/m}.$$

Since the line is lossless, Eq. (2.79) is valid:

$$\begin{aligned} Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) &= 50 \left[\frac{(40 + j20) + j50 \tan(2\pi \text{ rad/m} \times 2.5 \text{ m})}{50 + j(40 + j20) \tan(2\pi \text{ rad/m} \times 2.5 \text{ m})} \right] \\ &= 50 \left[(40 + j20) + j50 \times 0 \right] 50 + j(40 + j20) \times 0 \\ &= (40 + j20) \ \Omega. \end{aligned}$$

Problem 2.31 A voltage generator with

$$v_g(t) = 5 \cos(2\pi \times 10^9 t) \text{ V}$$

and internal impedance $Z_g = 50$ Ω is connected to a 50-Ω lossless air-spaced transmission line. The line length is 5 cm and the line is terminated in a load with impedance $Z_L = (100 - j100)$ Ω. Determine:

(a) Γ at the load.

(b) Z_{in} at the input to the transmission line.

(c) The input voltage $\widetilde{V}_i$ and input current $\widetilde{I}_i$.

(d) The quantities in (a)–(c) using CD Modules 2.4 or 2.5.

Solution:

(a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62e^{-j29.7^\circ}.$$

(b) All formulae for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9$ rad/s. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}.$$

Then, using Eq. (2.79),

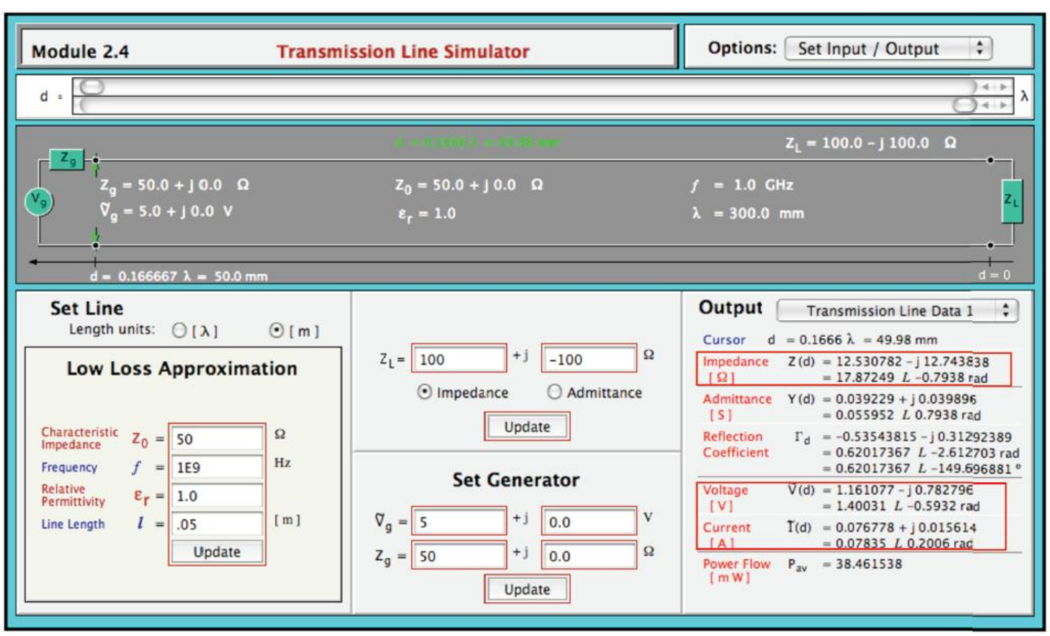
$$\begin{aligned}
Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \\
&= 50 \left[\frac{(100 - j100) + j50 \tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)}{50 + j(100 - j100) \tan\left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm}\right)} \right] \\
&= 50 \left[\frac{(100 - j100) + j50 \tan\left(\frac{\pi}{3} \text{ rad}\right)}{50 + j(100 - j100) \tan\left(\frac{\pi}{3} \text{ rad}\right)} \right] = (12.5 - j12.7)\ \Omega.
\end{aligned}$$

(c) In phasor domain, $\widetilde{V}_g = 5 \text{ V}e^{j0^\circ}$. From Eq. (2.80),

$$\widetilde{V}_i = \frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40e^{-j34.0^\circ} \quad \text{(V)},$$

and also from Eq. (2.80),

$$\widetilde{I}_i = \frac{\widetilde{V}_i}{Z_{in}} = \frac{1.4e^{-j34.0^\circ}}{(12.5 - j12.7)} = 78.4e^{j11.5^\circ} \quad \text{(mA)}.$$



Problem 2.33 Two half-wave dipole antennas, each with an impedance of 75 Ω, are connected in parallel through a pair of transmission lines, and the combination is connected to a feed transmission line, as shown in Fig. P2.33.

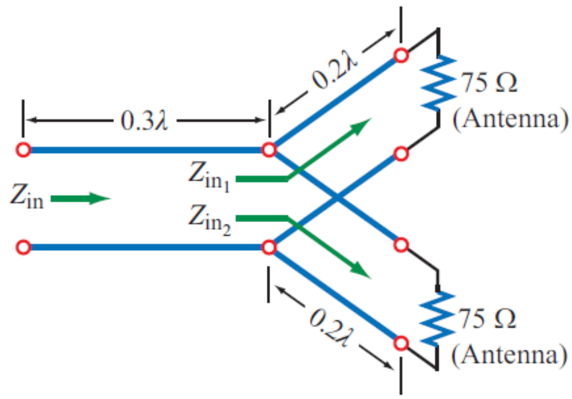


Figure P2.33: Circuit for Problem 2.33.

All lines are 50 Ω and lossless.

(a) Calculate Z_{in_1}, the input impedance of the antenna-terminated line, at the parallel juncture.

(b) Combine Z_{in_1} and Z_{in_2} in parallel to obtain Z'_L, the effective load impedance of the feedline.

(c) Calculate Z_{in} of the feedline.

Solution:

(a)

$$\begin{aligned} Z_{in_1} &= Z_0 \left[\frac{Z_{L_1} + jZ_0 \tan \beta l_1}{Z_0 + jZ_{L_1} \tan \beta l_1}\right] \\ &= 50 \left\{\frac{75 + j50 \tan[(2\pi/\lambda)(0.2\lambda)]}{50 + j75 \tan[(2\pi/\lambda)(0.2\lambda)]}\right\} = (35.20 - j8.62)\ \Omega. \end{aligned}$$

(b)

$$Z'_L = \frac{Z_{in_1} Z_{in_2}}{Z_{in_1} + Z_{in_2}} = \frac{(35.20 - j8.62)^2}{2(35.20 - j8.62)} = (17.60 - j4.31)\ \Omega.$$

(c)

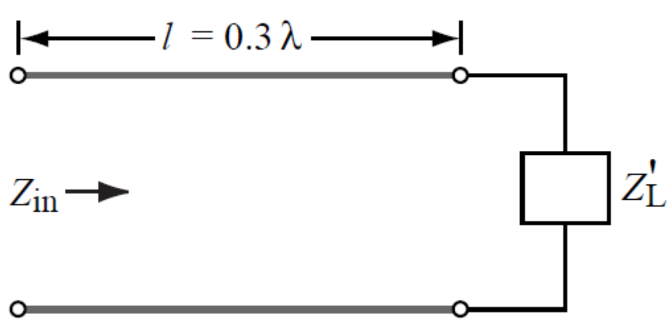


Figure P2.33: (b) Equivalent circuit.

$$Z_{in} = 50 \left\{\frac{(17.60 - j4.31) + j50 \tan[(2\pi/\lambda)(0.3\lambda)]}{50 + j(17.60 - j4.31) \tan[(2\pi/\lambda)(0.3\lambda)]}\right\} = (107.57 - j56.7)\ \Omega.$$

Problem 2.34 A 50-Ω lossless line is terminated in a load impedance $Z_L = (30 - j20)$ Ω.

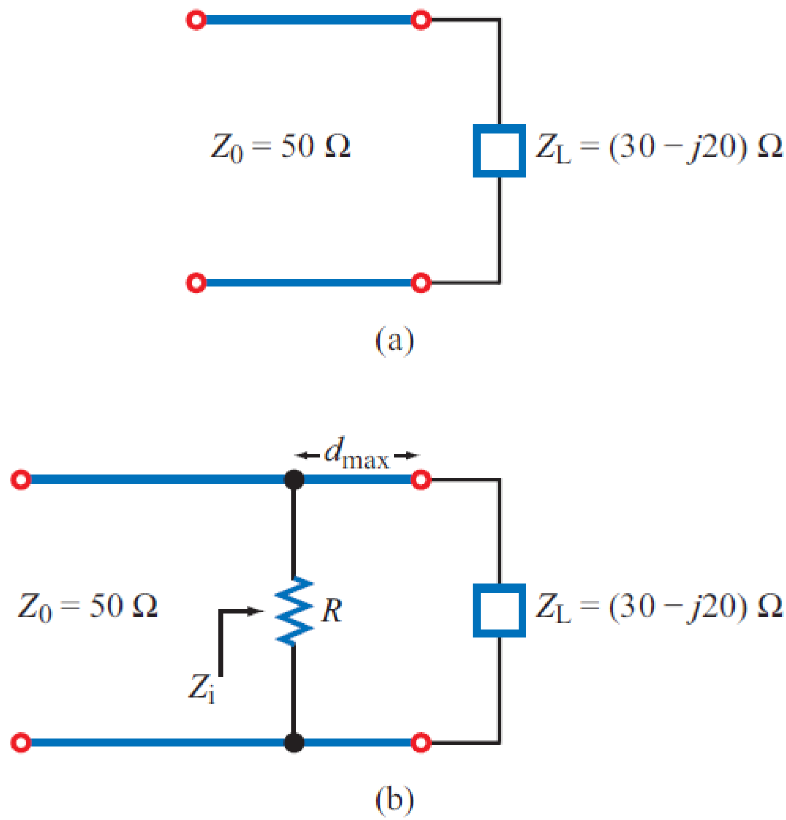


Figure P2.34: Circuit for Problem 2.34.

(a) Calculate Γ and S.

(b) It has been proposed that by placing an appropriately selected resistor across the line at a distance d_{max} from the load (as shown in Fig. P2.34(b)), where d_{max} is the distance from the load of a voltage maximum, then it is possible to render $Z_i = Z_0$, thereby eliminating reflection back to the end. Show that the proposed approach is valid and find the value of the shunt resistance.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j20 - 50}{30 - j20 + 50} = \frac{-20 - j20}{80 - j20} = \frac{-(20 + j20)}{80 - j20} = 0.34e^{-j121^\circ}.$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.34}{1 - 0.34} = 2.$$

(b) We start by finding d_{max}, the distance of the voltage maximum nearest to the load. Using (2.70) with $n = 1$,

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} = \left(\frac{-121^\circ \pi}{180^\circ}\right)\frac{\lambda}{4\pi} + \frac{\lambda}{2} = 0.33\lambda.$$

Applying (2.79) at $d = d_{max} = 0.33\lambda$, for which $\beta l = (2\pi/\lambda) \times 0.33\lambda = 2.07$ radians, the value of Z_{in} before adding the shunt resistance is:

$$\begin{aligned} Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right) \\ &= 50 \left(\frac{(30 - j20) + j50 \tan 2.07}{50 + j(30 - j20) \tan 2.07} \right) = (102 + j0)\ \Omega. \end{aligned}$$

Thus, at the location A (at a distance d_{max} from the load), the input impedance is purely real. If we add a shunt resistor R in parallel such that the combination is equal to Z_0, then the new Z_{in} at any point to the left of that location will be equal to Z_0.

Hence, we need to select R such that

$$\frac{1}{R} + \frac{1}{102} = \frac{1}{50}$$

or $R = 98\ \Omega$.

Problem 2.40 A 100-MHz FM broadcast station uses a 300-Ω transmission line between the transmitter and a tower-mounted half-wave dipole antenna. The antenna impedance is 73 Ω. You are asked to design a quarter-wave transformer to match the antenna to the line.

(a) Determine the electrical length and characteristic impedance of the quarter-wave section.

(b) If the quarter-wave section is a two-wire line with $D = 2.5$ cm, and the wires are embedded in polystyrene with $\varepsilon_{\mathrm{r}} = 2.6$, determine the physical length of the quarter-wave section and the radius of the two wire conductors.

Solution:

(a) For a match condition, the input impedance of a load must match that of the transmission line attached to the generator. A line of electrical length $\lambda/4$ can be used. From Eq. (2.97), the impedance of such a line should be

$$Z_0 = \sqrt{Z_{\mathrm{in}} Z_{\mathrm{L}}} = \sqrt{300 \times 73} = 148\ \Omega.$$

(b)

$$\frac{\lambda}{4} = \frac{u_{\mathrm{p}}}{4f} = \frac{c}{4\sqrt{\varepsilon_{\mathrm{r}}}f} = \frac{3 \times 10^8}{4\sqrt{2.6} \times 100 \times 10^6} = 0.465\ \mathrm{m},$$

and, from Table 2-2,

$$Z_0 = \frac{120}{\sqrt{\varepsilon}} \ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right]\ \Omega.$$

Hence,

$$\ln\left[\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1}\right] = \frac{148\sqrt{2.6}}{120} = 1.99,$$

which leads to

$$\left(\frac{D}{d}\right) + \sqrt{\left(\frac{D}{d}\right)^2 - 1} = 7.31,$$

and whose solution is $D/d = 3.73$. Hence, $d = D/3.73 = 2.5\ \mathrm{cm}/3.73 = 0.67$ cm.

Problem 2.44 For the circuit shown in Fig. P2.44, calculate the average incident power, the average reflected power, and the average power transmitted into the infinite 100-Ω line. The $\lambda/2$ line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as $\alpha \neq 0$.)

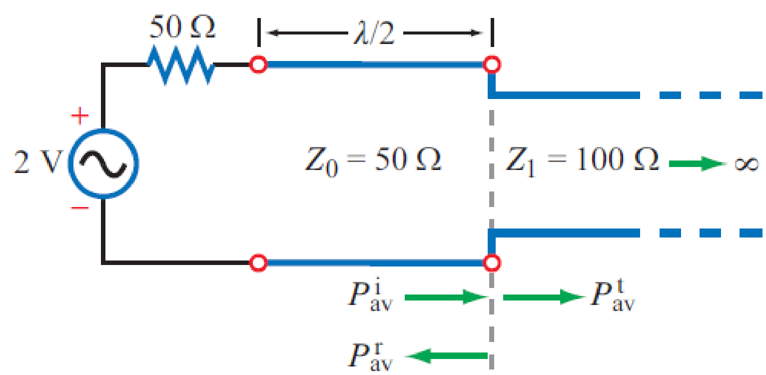


Figure P2.44: Circuit for Problem 2.44.

Solution: Considering the semi-infinite transmission line as equivalent to a load (since all power sent down the line is lost to the rest of the circuit), $Z_L = Z_1 = 100\ \Omega$. Since the feed line is $\lambda/2$ in length, Eq. (2.96) gives $Z_{in} = Z_L = 100\ \Omega$ and $\beta l = (2\pi/\lambda)(\lambda/2) = \pi$, so $e^{\pm j\beta l} = -1$. Hence

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100-50}{100+50} = \frac{1}{3}.$$

Also, converting the generator to a phasor gives $\widetilde{V}_g = 2e^{j0^\circ}$ (V). Plugging all these results into Eq. (2.82),

$$V_0^+ = \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{e^{j\beta l} + \Gamma e^{-j\beta l}}\right) = \left(\frac{2\times 100}{50+100}\right)\left[\frac{1}{(-1)+\frac{1}{3}(-1)}\right]$$
$$= 1e^{j180^\circ} = -1 \quad \text{(V)}.$$

From Eqs. (2.104), (2.105), and (2.106),

$$P_{av}^i = \frac{|V_0^+|^2}{2Z_0} = \frac{|1e^{j180^\circ}|^2}{2\times 50} = 10.0 \text{ mW},$$

$$P_{av}^r = -|\Gamma|^2 P_{av}^i = -\left|\frac{1}{3}\right|^2 \times 10 \text{ mW} = -1.1 \text{ mW},$$

$$P_{av} = P_{av}^t = P_{av}^i + P_{av}^r = 10.0 \text{ mW} - 1.1 \text{ mW} = 8.9 \text{ mW}.$$

Problem 2.47 Use the Smith chart to find the reflection coefficient corresponding to a load impedance of

(a) $Z_L = 3Z_0$

(b) $Z_L = (2 - j2)Z_0$

(c) $Z_L = -j2Z_0$

(d) $Z_L = 0$ (short circuit)

Solution: Refer to Fig. P2.47.

(a) Point A is $z_L = 3 + j0$. $\Gamma = 0.5e^{0^\circ}$

(b) Point B is $z_L = 2 - j2$. $\Gamma = 0.62e^{-29.7^\circ}$

(c) Point C is $z_L = 0 - j2$. $\Gamma = 1.0e^{-53.1^\circ}$

(d) Point D is $z_L = 0 + j0$. $\Gamma = 1.0e^{180.0^\circ}$

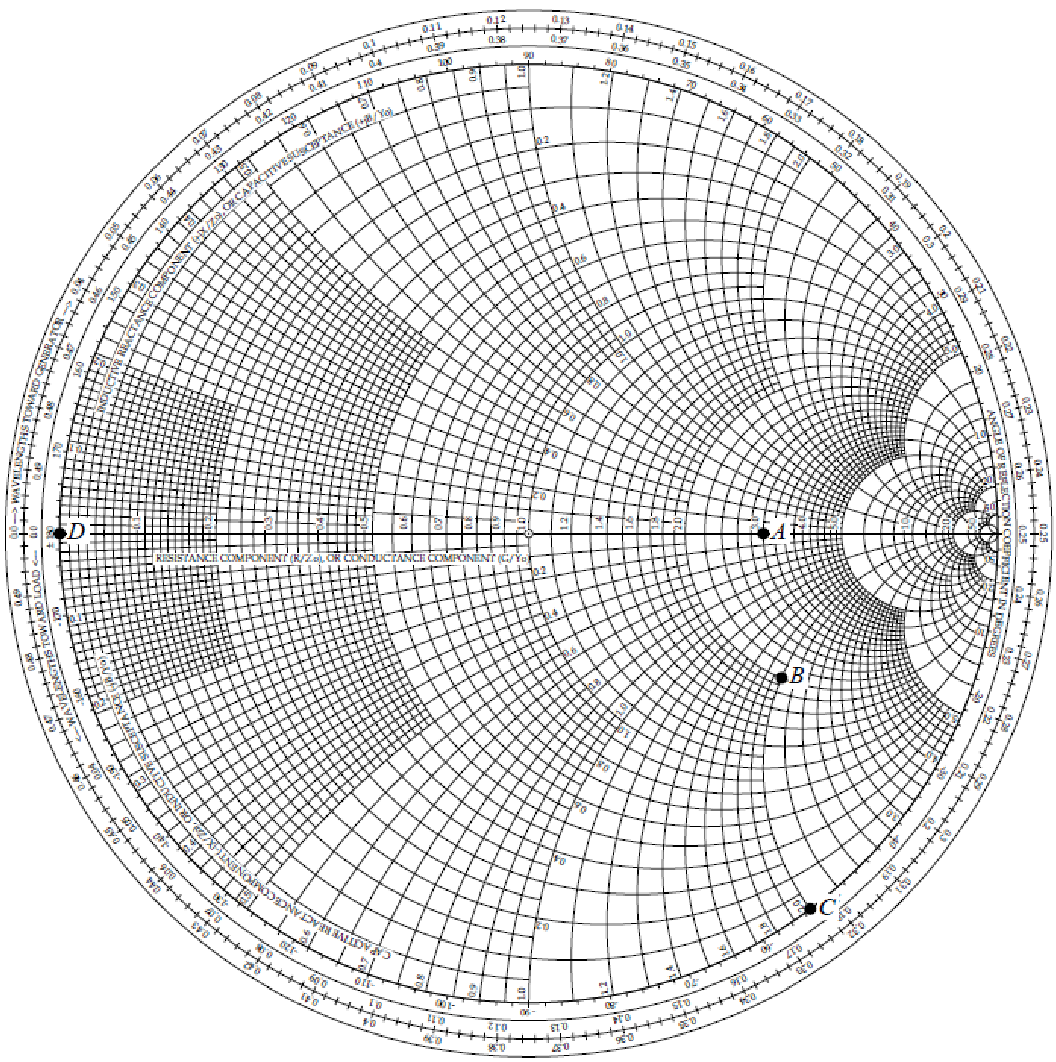


Figure P2.47: Solution of Problem 2.47.

Problem 2.53 A lossless 50-Ω transmission line is terminated in a load with $Z_L = (50 + j25)\ \Omega$. Use the Smith chart to find the following:

(a) The reflection coefficient Γ.

(b) The standing-wave ratio.

(c) The input impedance at 0.35λ from the load.

(d) The input admittance at 0.35λ from the load.

(e) The shortest line length for which the input impedance is purely resistive.

(f) The position of the first voltage maximum from the load.

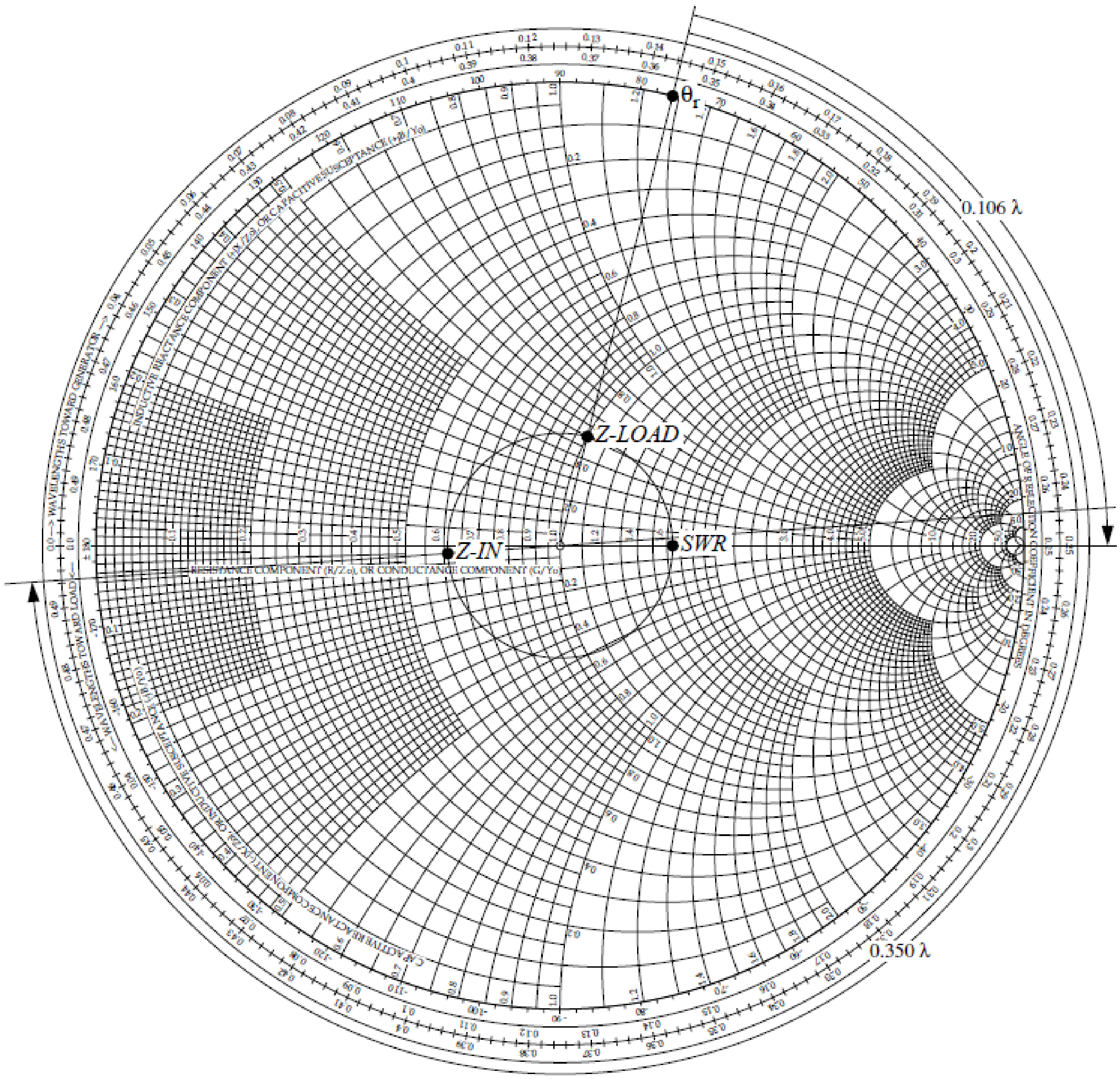


Figure P2.53: Solution of Problem 2.53.

Solution: Refer to Fig. P2.53. The normalized impedance

$$z_L = \frac{(50 + j25)\ \Omega}{50\ \Omega} = 1 + j0.5$$

is at point *Z-LOAD*.

(a) $\Gamma = 0.24e^{j76.0^\circ}$ The angle of the reflection coefficient is read of that scale at the point θ_r.

(b) At the point *SWR*: $S = 1.64$.

(c) Z_{in} is 0.350λ from the load, which is at 0.144λ on the wavelengths to generator scale. So point *Z-IN* is at $0.144\lambda + 0.350\lambda = 0.494\lambda$ on the WTG scale. At point *Z-IN*:

$$Z_{in} = z_{in}Z_0 = (0.61 - j0.022) \times 50\ \Omega = (30.5 - j1.09)\ \Omega.$$

(d) At the point on the SWR circle opposite *Z-IN*,

$$Y_{in} = \frac{y_{in}}{Z_0} = \frac{(1.64 + j0.06)}{50\ \Omega} = (32.7 + j1.17)\ \text{mS}.$$

(e) Traveling from the point *Z-LOAD* in the direction of the generator (clockwise), the SWR circle crosses the $x_L = 0$ line first at the point *SWR*. To travel from *Z-LOAD* to *SWR* one must travel $0.250\lambda - 0.144\lambda = 0.106\lambda$. (Readings are on the wavelengths to generator scale.) So the shortest line length would be 0.106λ.

(f) The voltage max occurs at point *SWR*. From the previous part, this occurs at $z = -0.106\lambda$.